Algebra with Legos™, Spreadsheets, and Dynamic Geometry Software

Susan Addington
Math Department, Cal State San Bernardino
saddingt@csusb.edu
http://www.quadrivium.info

Outline of session

1. Build the addition and multiplication tables with Legos

Discussion questions:

a. How is the multiplication graph different from the addition graph?

b. Can you describe the (top) surface of each graph?

c. Describe the cross sections of each graph along the rows (or columns) of the original table as a 2D graph.

d. What about other vertical cross sections of the graphs? Are they all lines?

e. Describe the horizontal cross sections of the graphs. Are they all lines?

2. Demonstration of using a spreadsheet to build these tables, then view them as several types of graphs

a. 2D contour graph of the 3D graph

b. Relate to the original table, with cells colored according to some rule. Example: all cells of the addition table with sum 7; all cells of the multiplication table with product 12

c. Interpolation and continuity: The Lego graphs have places only for integer values. In a table you can leave spaces and fill in in-between values. In a contour graph, you can fill in *all* in-between values, such as all pairs (x,y) with x+y=7.

3. Classification of many traditional word problems as "like the addition table" (linear function of 2 variables), or "like the multiplication table" (bilinear form in 2 variables); solution with dynamic geometry software

a. The product of two numbers is 48; their sum is 14. What are the numbers?

b. A rectangle has area 12 cm and perimeter 20 cm. What are its dimensions?

c. Pencils cost 12 cents apiece; erasers cost 28 cents. Mimi bought 7 of these items for $1. How many of each did she buy?

d. Raisins cost $1.89 per pound; peanuts cost $2.47 per pound. Sam bought 1 pound of raisins and peanuts for trail mix and paid $2.15. How much of each ingredient did he buy?

e. 3 pencils and 5 erasers cost 92 cents. 2 pencils and 6 erasers cost 96 cents. How much will 4 pencils and 3 erasers cost?

f. It takes Hiep 40 minutes to clean up after dinner. His sister takes 60 minutes to do the same job. How long will it take if they work together?

g. (A problem from 100 years ago) If 18 men consume 34 barrels of potatoes in 135 days, how long will it take 45 men to consume 102 barrels?

h. A hen and a half can lay an egg and a half in a day and a half. How long will it take 15 hens to lay 20 eggs? How many hens will it take to lay 6 eggs in 2 days? How many eggs can 3 hens lay in 5 days?

i. A woman can make a baby in 9 months. How long will it take 5 women to make 7 babies?

Note: many word problems, such as "two trains" problems, can be solved with two functions of one variable (as opposed to functions of two variables, as discussed in this session.)

Math content

This presentation is based on activities from the Algebraic Thinking section of *Measuring the World: Mathematics for Elementary and Middle School* Teachers. A preliminary version appeared in the Linearity module from the professional development curriculum *Ways to Think About Mathematics*.

Legos

To make both the addition and multiplication table at the same time, you need:

1500 2x2 bricks (part # 3003) (this number includes plenty of extras)
240 2x2 plates (part # 3022) (this number includes plenty of extras)
2 32x32 baseplates (part # 3811); you can buy these at Toys R Us
2 brick separators (no part number. These will make your life easier.)

Warning! You may have large quantities of Legos at home. You will be unpleasantly surprised at how few of them are 2x2 bricks and plates, and it will take days of your time to sort them all out. Instead, buy a new set.

The most economical way to get exactly what you need is http://www.bricklink.com. Bricklink is a site for people who sell Legos, mostly obtained by sorting parts from new or used sets. The seller with the largest quantity, best prices, and efficient service is Precious Princess Palace. If you buy from her, the bricks will cost about $30.

You can also buy individual Legos at lego.com, but they cost more (10 cents at Lego vs. 1.2 cents per brick at Precious Princess.) It is easier to order on the Lego site, though.

Software

Microsoft Excel. The standard spreadsheet program, part of MS Office. Costs money.

OpenOffice.org. (http://www.OpenOffice.org) A free, open-source package designed to act like, and be compatible with, Microsoft Office. Includes a spreadsheet, word

processor with adequate mathematical symbol formatter, presentation software, a good drawing program and database. OpenOffice Spreadsheet does not (yet?) do surface or contour graphs, and formatting is a little less convenient than Excel.

Google Docs (http://docs.google.com) has a spreadsheet. Your files and the software are on Google's server, and you can grant access to others to view or work on the document. Requires a free Google account (but you sacrifice some personal information.)

GeoGebra, http://www.geogebra.org. Free dynamic 2D geometry software that can be downloaded or run over the web. Does an excellent job of integrating algebra and geometry. Includes an integrated spreadsheet.

Graphing Calculator (the name of the software, not a hardware device) Purchase by download only from http://www.PacificT.com. An educational copy costs $60. Easy to use, does many kinds of 2D, 3D, and 4D graphs. Excellent graphics.

How to use software

A short introduction to math with spreadsheets is at
http://www.quadrivium.info/MtW/A2.Spreadsheets.pdf

References

Addington, Susan, and David Dennis. Measuring the World: Mathematics for Elementary and Middle School Teachers. Private website; account required. Please ask for access. http://www.csusb-odl.com/moodle/course/view.php?id=9.

Benson, Steve, Susan Addington, Nina Arshavsky, Al Cuoco, E. Paul Goldenberg, and Eric Karnowski. 2004. Ways to Think About Mathematics: Activities and Investigations for Grade 6-12 Teachers. Corwin Press.

Driscoll, Mark. 1999. Fostering Algebraic Thinking: A Guide for Teachers, Grades 6-10. Heinemann.

Gerofsky, Susan. 2002. A Man Left Albuquerque Heading East: Word Problems As Genre in Mathematics Education. Peter Lang Publishing. A somewhat subversive treatment of story problems as actual stories.

Answers

a. 6 and 8 b. 8.61 cm by 1.39 cm (to nearest 0.01 cm) c. 6 pencils and 1 eraser

d. 0.55 lb. of raisins, 0.45 lb. of peanuts (to nearest 0.01 lb.) e. 75 cents f. 24 minutes

g. 162 days h. 2 days; 4 $\frac{1}{2}$ hens; 10 eggs

i. This seems to be very like the hens problem. Is it?

Which were easiest to solve with algebra? With graphs? With tables? By just guessing? Also try dimensional analysis (computations with units.) Was it the problem or the particular numbers that made one method easier?